Bubble propagation in Hele-Shaw channels with centred constrictions

A. Franco-Gómez, A.L. Hazel, A.B. Thompson & A. Juel
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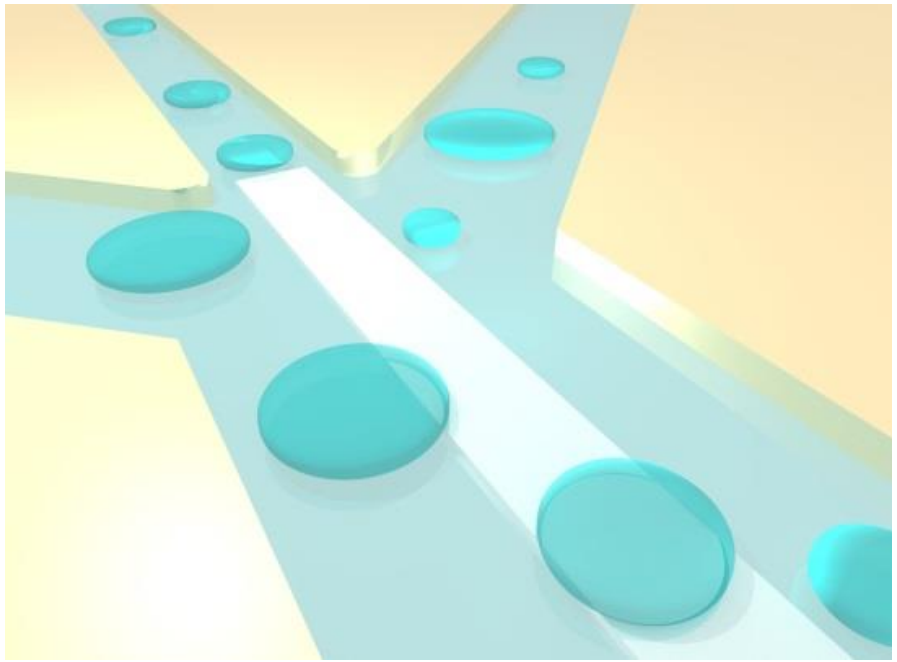
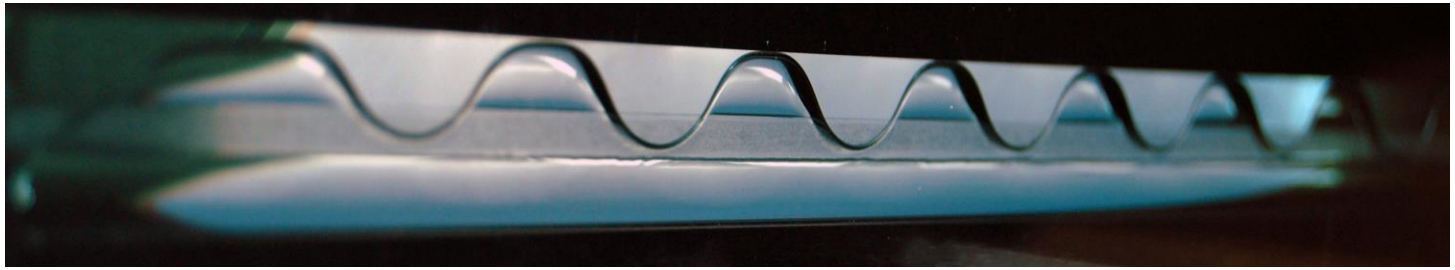
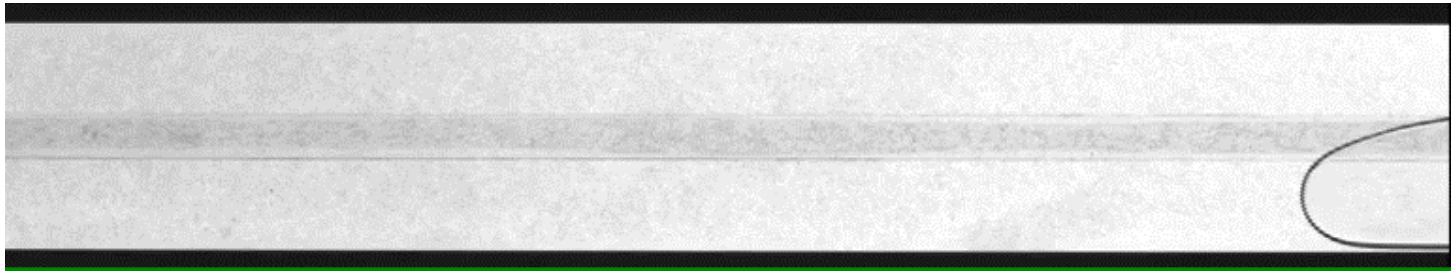
University of Manchester







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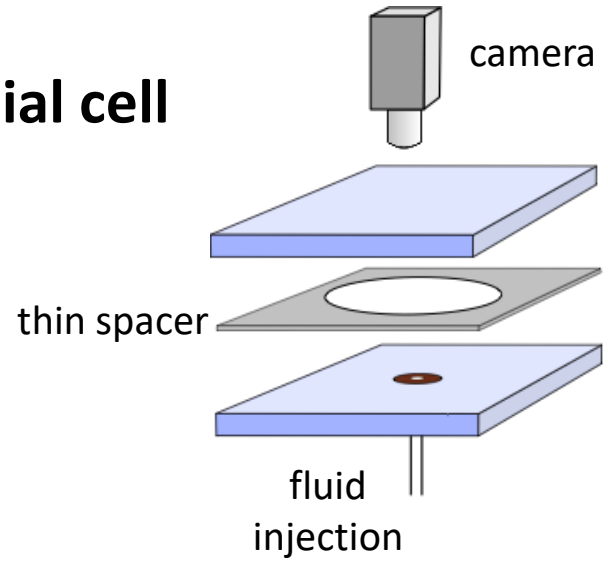
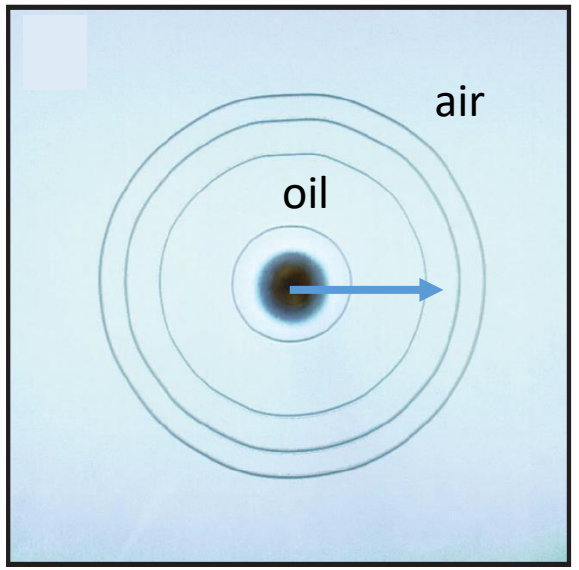
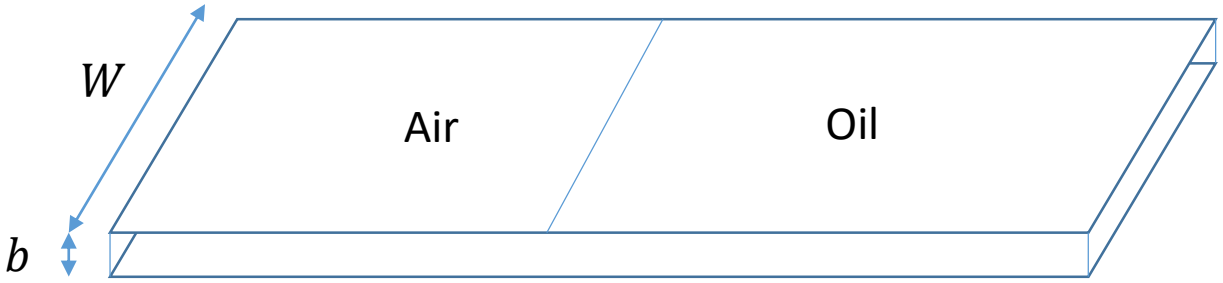
A canonical system: Displacement flow in a Hele-Shaw cell

Radial cell





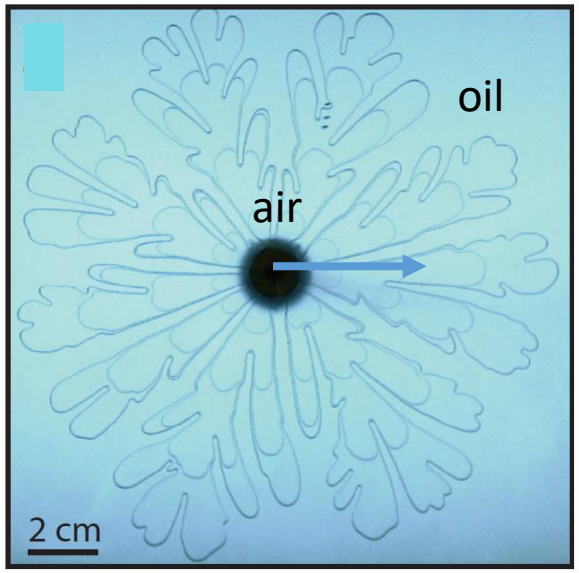
Channel



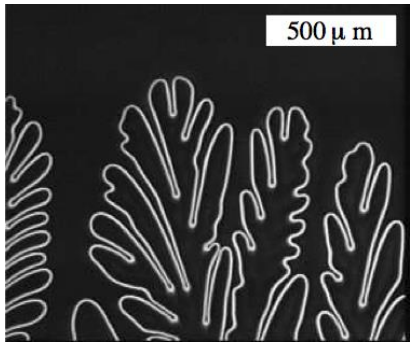
$$\alpha = \frac{W}{b} \gg 1$$

- Quasi 2D
- Model for porous media
- Pair of non-wetting and wetting fluids.

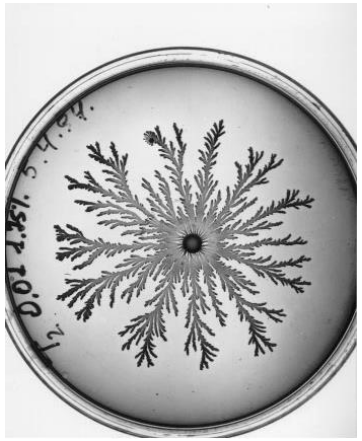
Unstable front propagation



Canonical model

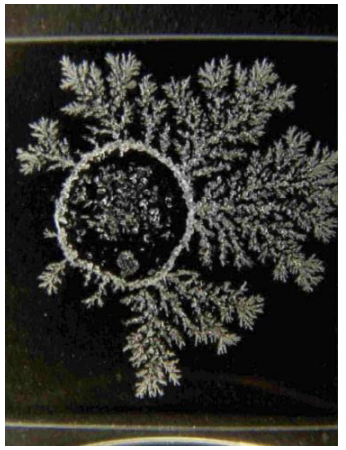


Solidification



Bacterial colonies

Electrodeposition

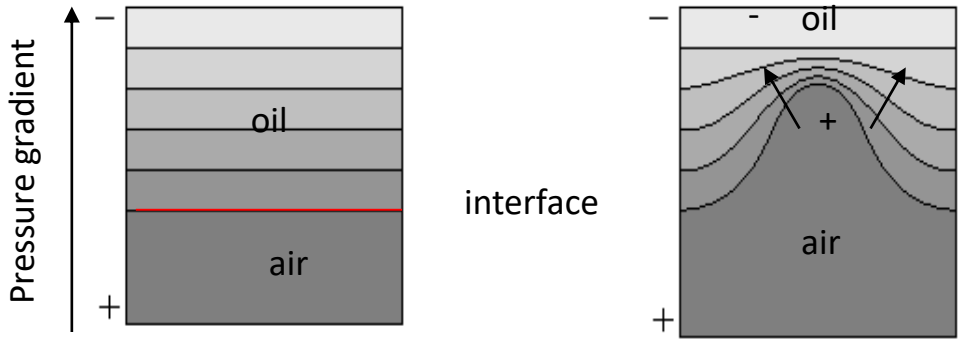


Drying

Saffman-Taylor instability

Saffman & Taylor 1958

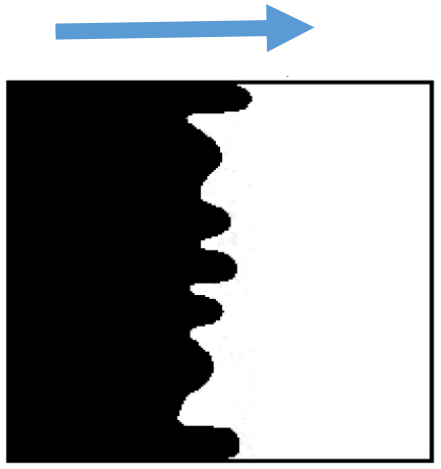
Instability mechanism:



- Applied pressure gradient works against viscous and surface tension forces.
- Key parameter:

$$Ca = \frac{\text{viscous forces}}{\text{surface tension forces}}$$

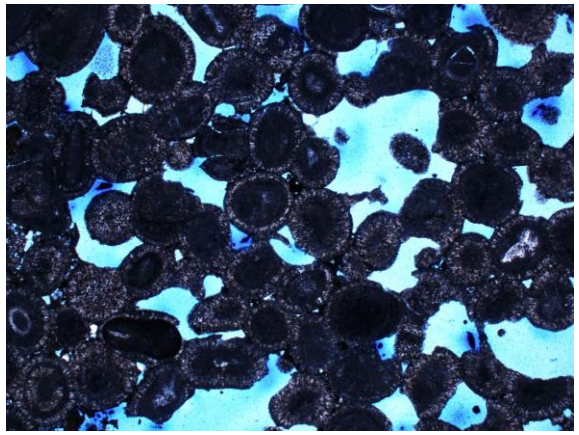
Channel



Fingers grow and compete

Single stable finger

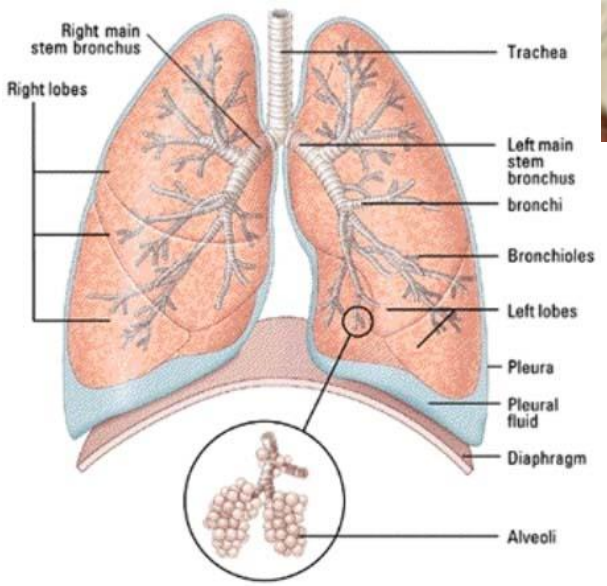
Applications of displacement flows

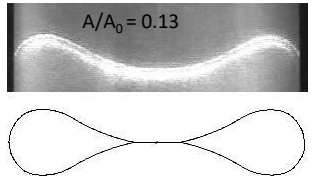


Complex pore shapes (light) in 2D slices of carbonates

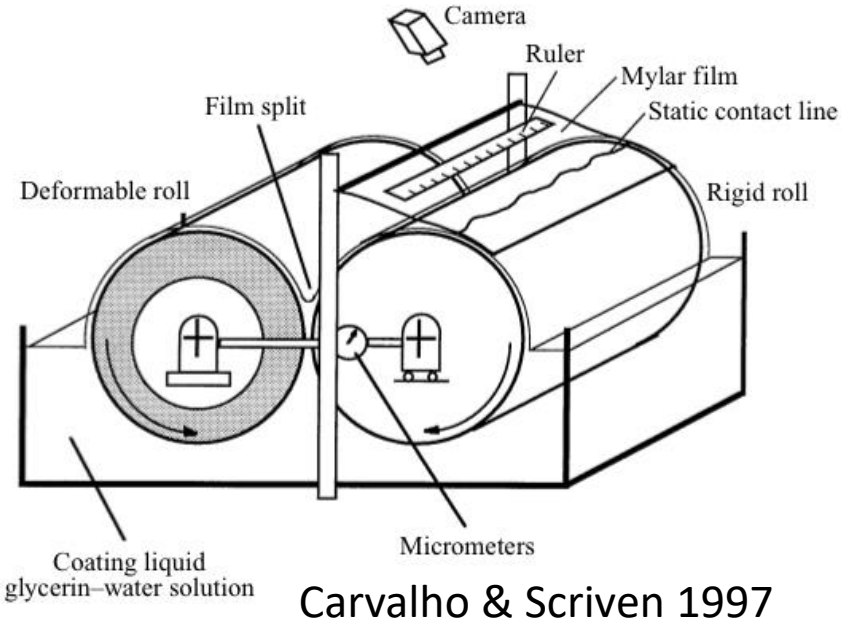
Porous media

Pulmonary airway reopening





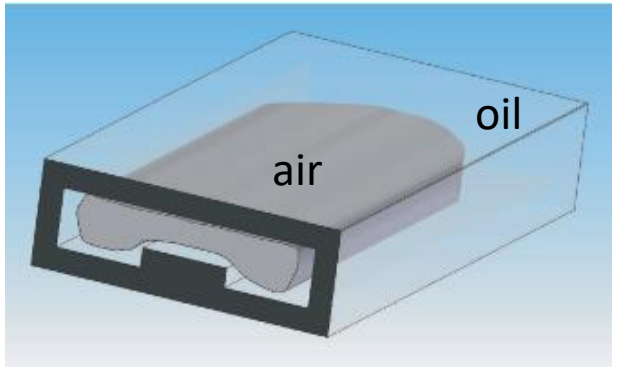
Industrial processes:



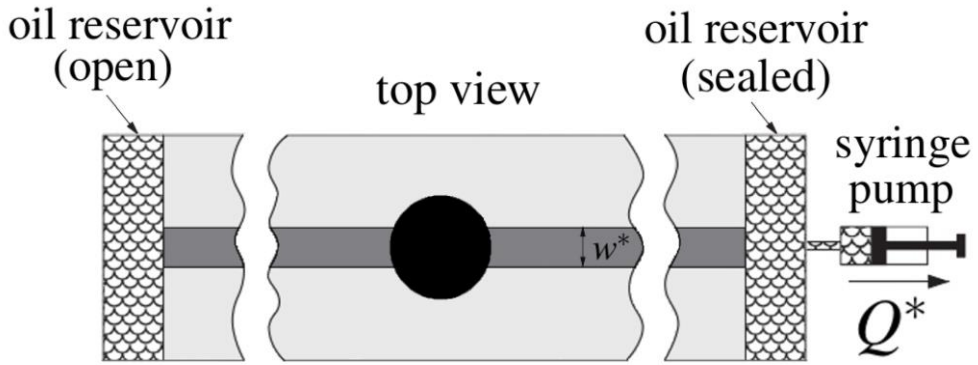
Carvalho & Scriven 1997

Cleaning and decontamination?

- Complex vessel geometry: focus on Hele-Shaw cell of variable depth.

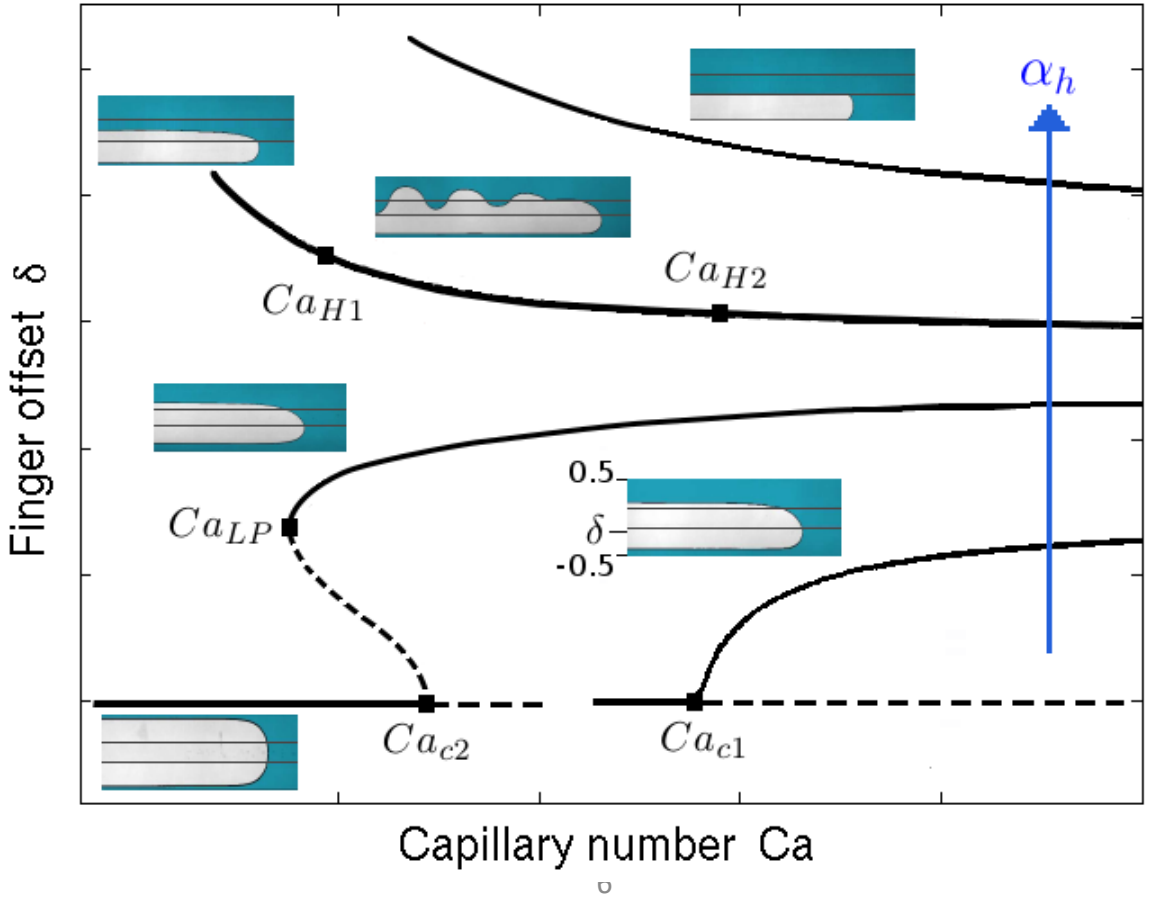


Rigid Hele-Shaw channel with depth variation



- Vary α_h (height of constriction).
- Constriction height could be less than roughness of channel boundaries.

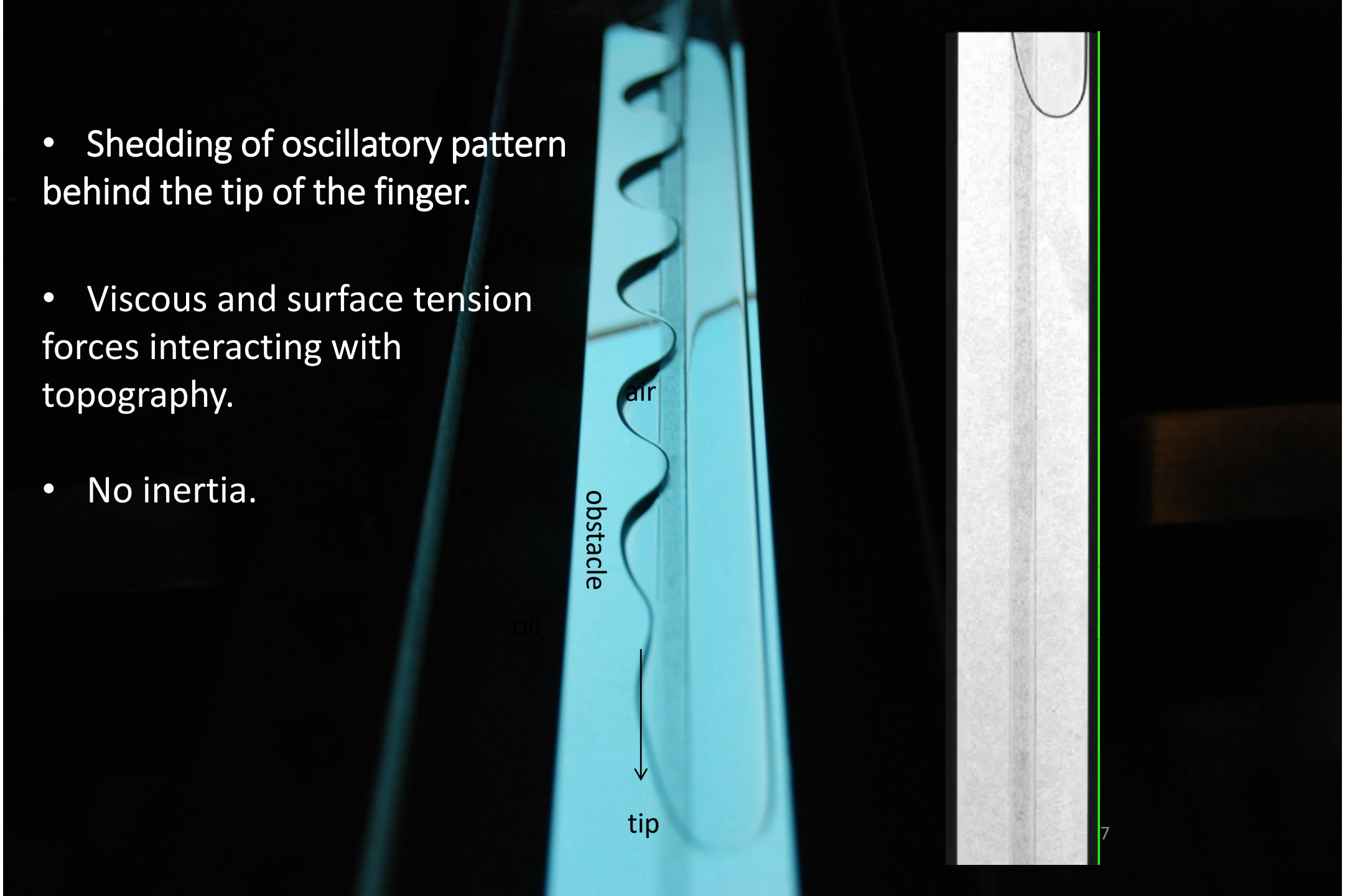
With the constriction



- Shedding of oscillatory pattern behind the tip of the finger.

- Viscous and surface tension forces interacting with topography.

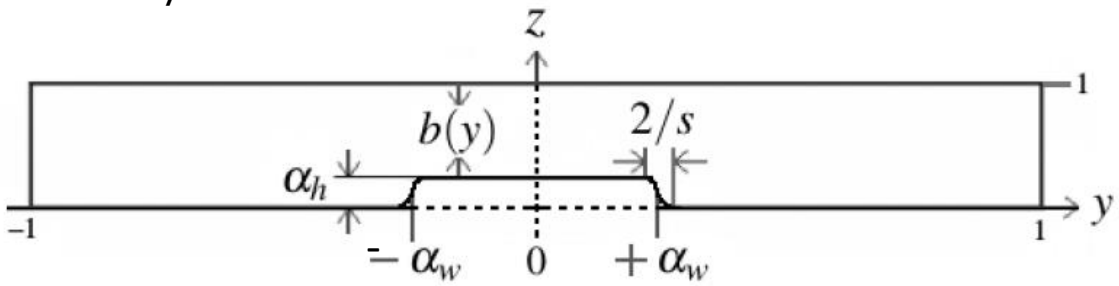
- No inertia.



Depth-averaged model

Thompson, Juel & Hazel JFM 2014

- Aspect ratio: $\alpha = W/H \gg 1$



- Depth averaging reduces 3D Stokes to 2D Hele–Shaw eqns, with $b = b(y)$,

$$\nabla_H . (b^3 \nabla_H p) = 0$$

- On bubble boundary,

$$\boldsymbol{R}_t . \boldsymbol{n} = \boldsymbol{u} . \boldsymbol{n} \quad \text{(kinematic condition)},$$

$$p_{bubble} - p = \frac{1}{3\alpha Q}\left(\frac{2}{b(y)} + \frac{\kappa_{\parallel}}{\alpha}\right),$$

(normal stress balance),

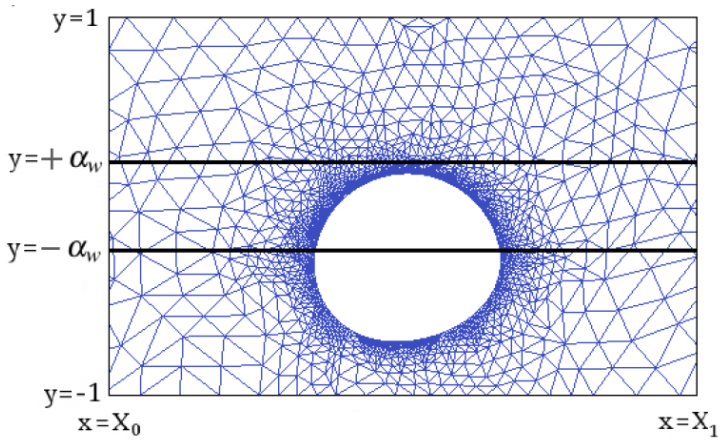
$Q = Ca/U_f$ (non-dimensional flow rate)

- Solved numerically: oomph-lib

the object-oriented multi-physics finite-element library

www.oomph-lib.org

- Extending model of McLean & Saffman JFM (1981)

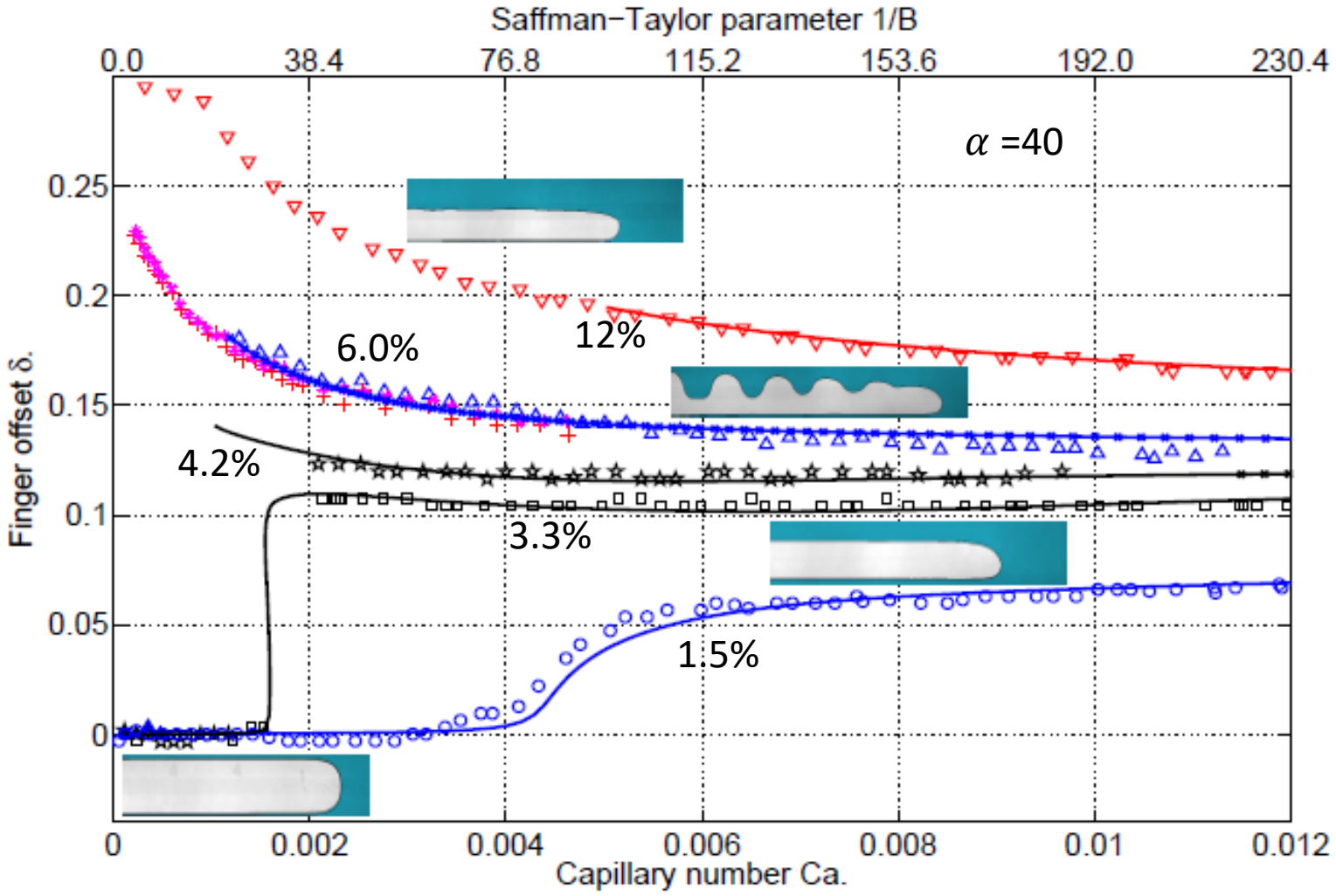


Problems with depth averaged model

- Cannot impose no slip boundary condition, only non-penetration.
- Ignores thin-films above and below bubble, which influence mass conservation and bubble shape.
- Ignores correction to curvature in the static limit to account for “pancake" shape of the bubble.

.... and yet!

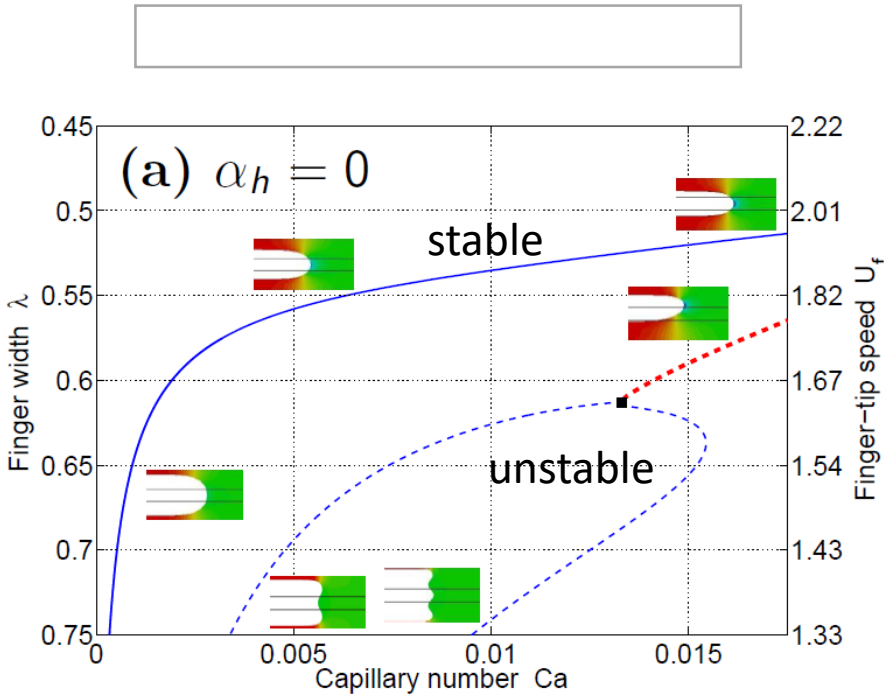
Predictive model for $\alpha \geq 40$

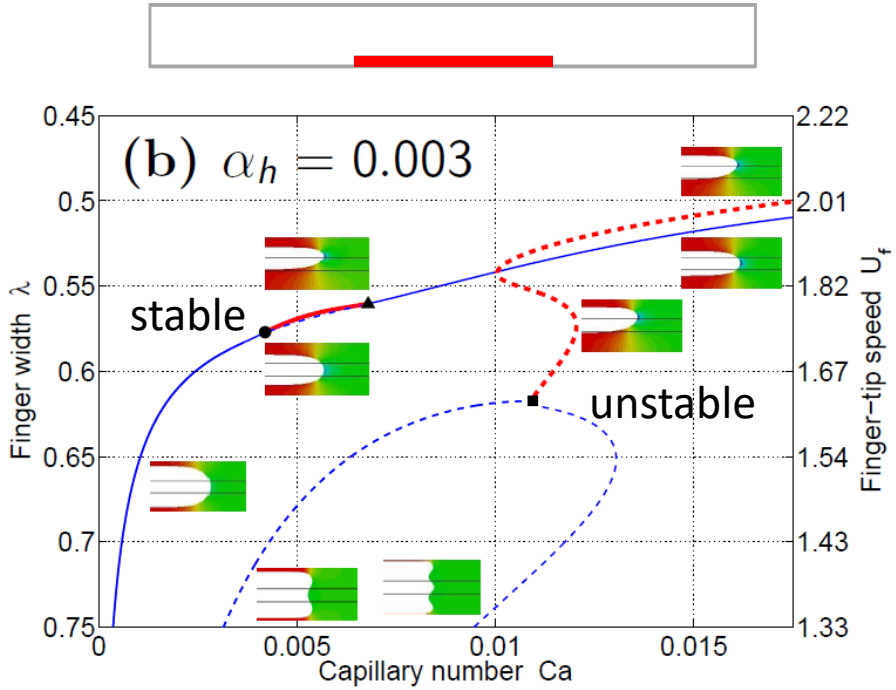


Franco-Gómez, Thompson, Hazel & Juel JFM 2016

Steady solutions

α = 80

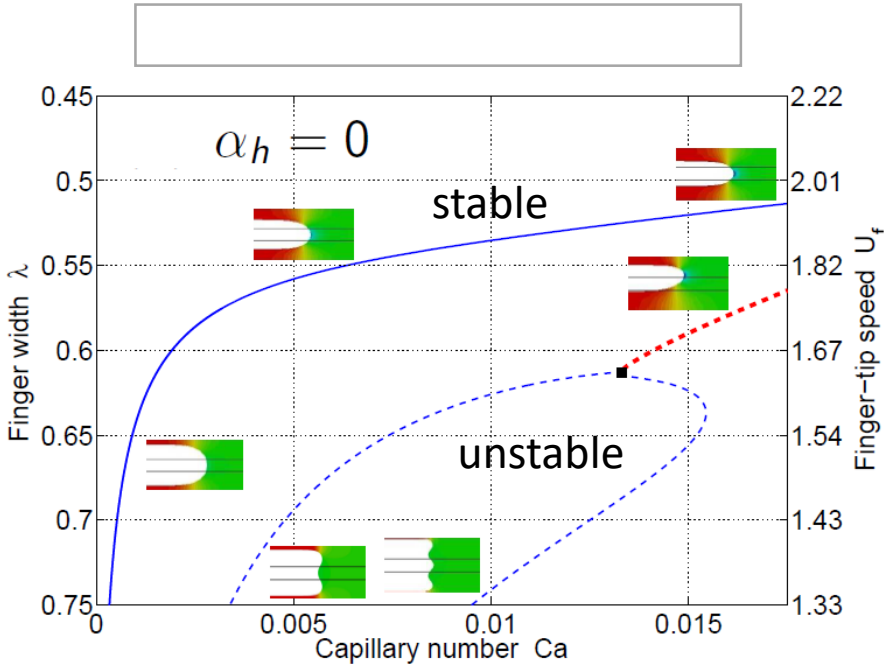


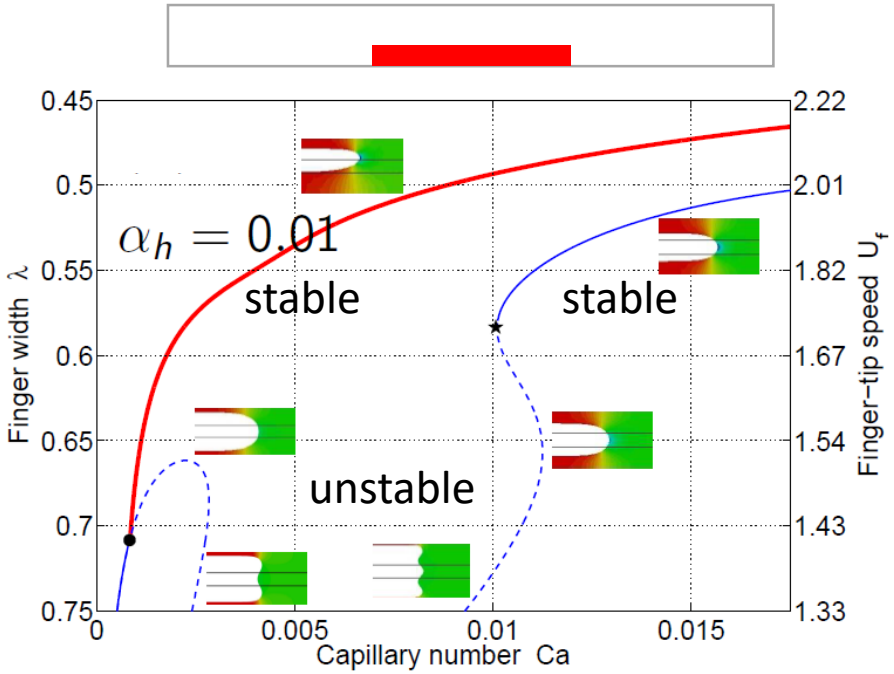


- In fact, in the absence of rail, multiple steady solutions, but only one stable.
- Multiplicity of solutions enables alteration of bifurcation structure.
- Small but finite rail height required for multiple stable solutions to emerge.

Steady solutions

α = 80



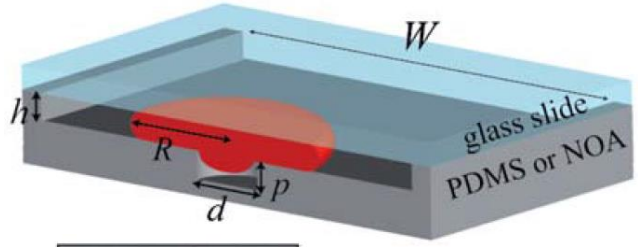


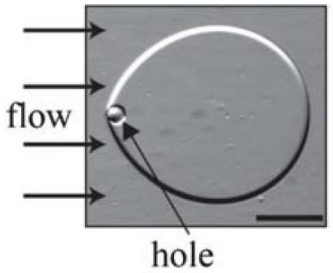
Smaller occlusion heights required for increasing α.

- Symmetry breaking: $\alpha_h \sim Ca^{-1}\alpha^{-2}$
- Hopf bifurcation: $\alpha_h \sim \alpha^{-1}$

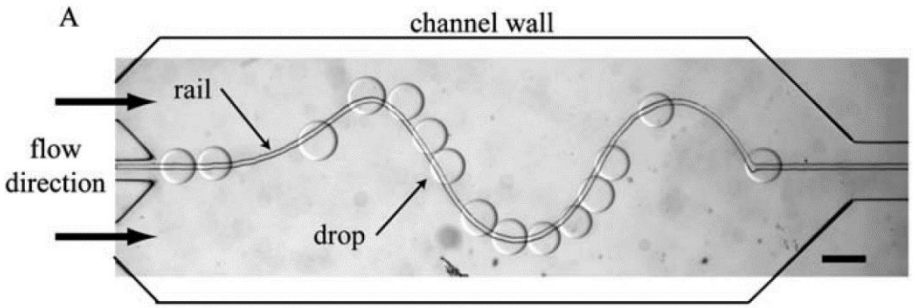
Application: A concept for sorting bubbles by size.

Rails and anchors: guiding and trapping droplets in 2D microreactors.



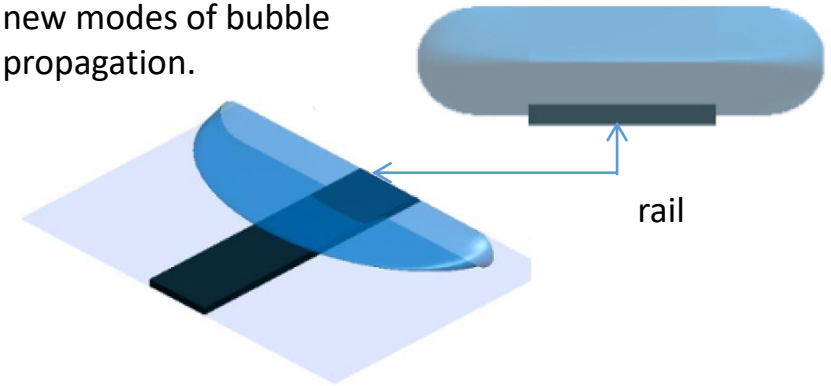


Abbyad et al. 2011
Lab Chip 11, 813-821.

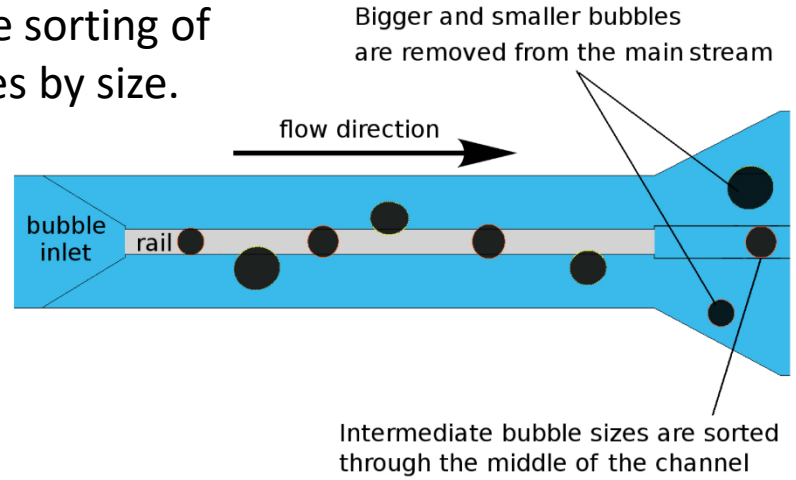


Static: modification of capillary-static mode.

We locally reduce the depth of the channel to create new modes of bubble propagation.



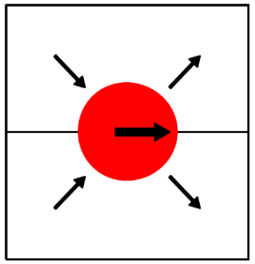
Passive sorting of bubbles by size.



Dynamic interaction between viscous and surface tension forces.

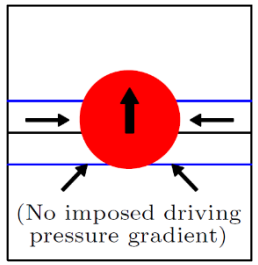
Off-rail bubbles at rest

Without rail, single mode of propagation.



With rail, cross-sectional curvature increased on rail.

Focus on small bubbles (size of the rail)



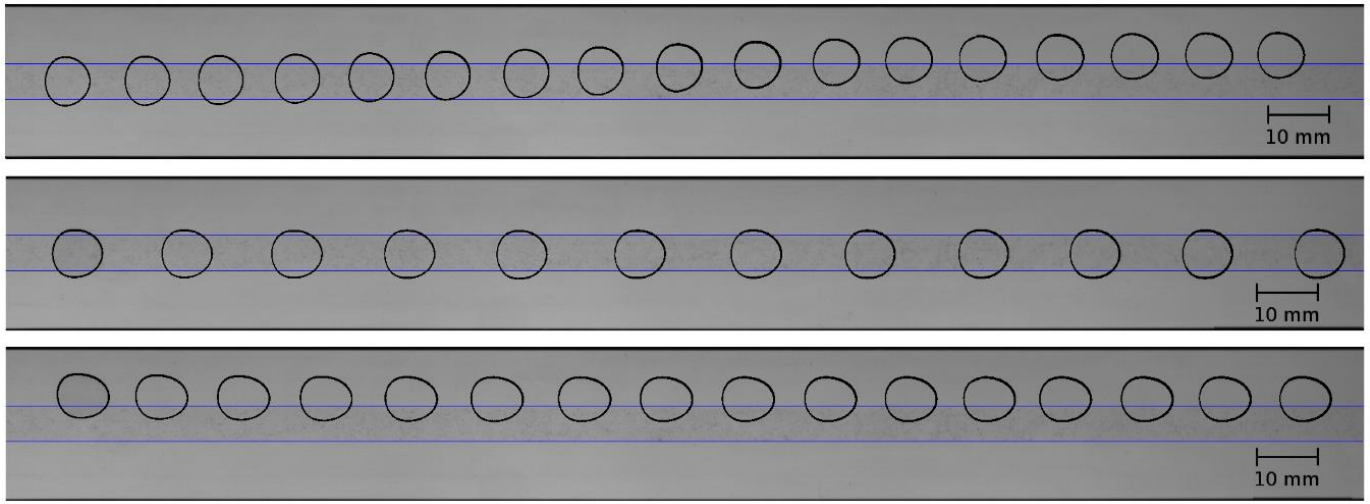
- Laplace equation (normal stress):

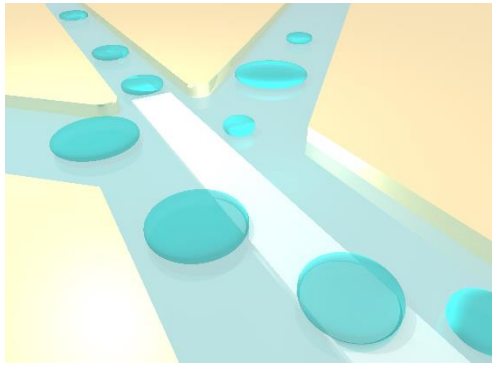
$$p = p_b - \sigma(\kappa_{\parallel} + \kappa_{\perp})$$

- Lowers pressure in fluid on rail near bubble and induces pressure gradient to push ***small*** bubble off rail in the absence of imposed flow (capillary-static).

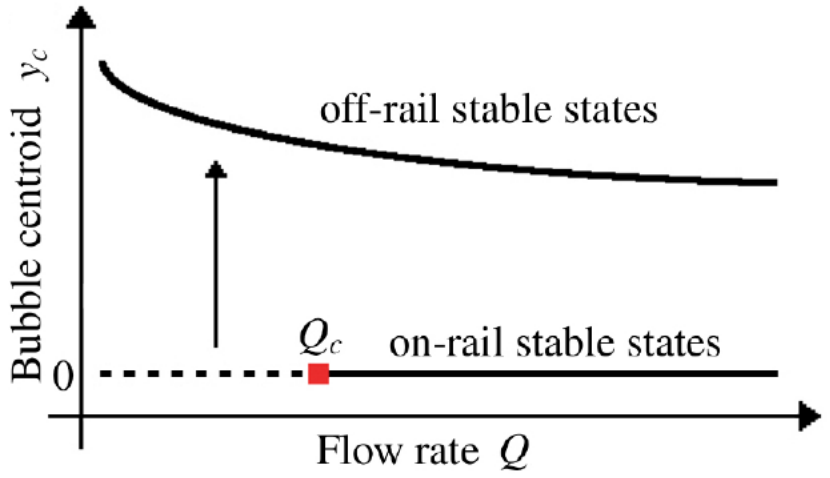
Can interaction between viscous and surface tension forces stabilise on-rail bubbles dynamically?

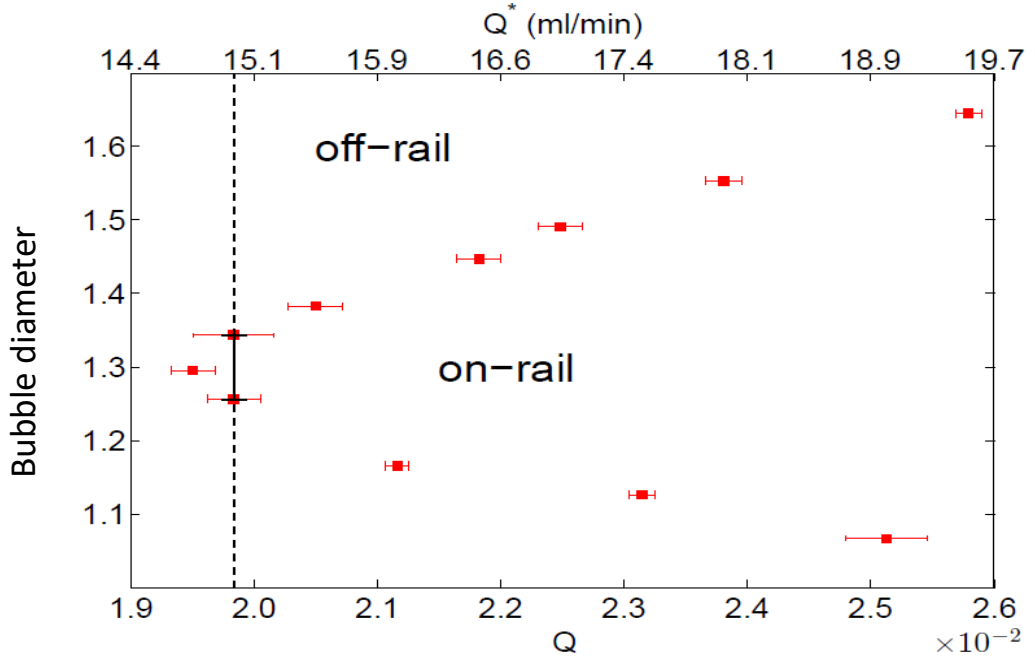
Tunable on-rail bubble sizes



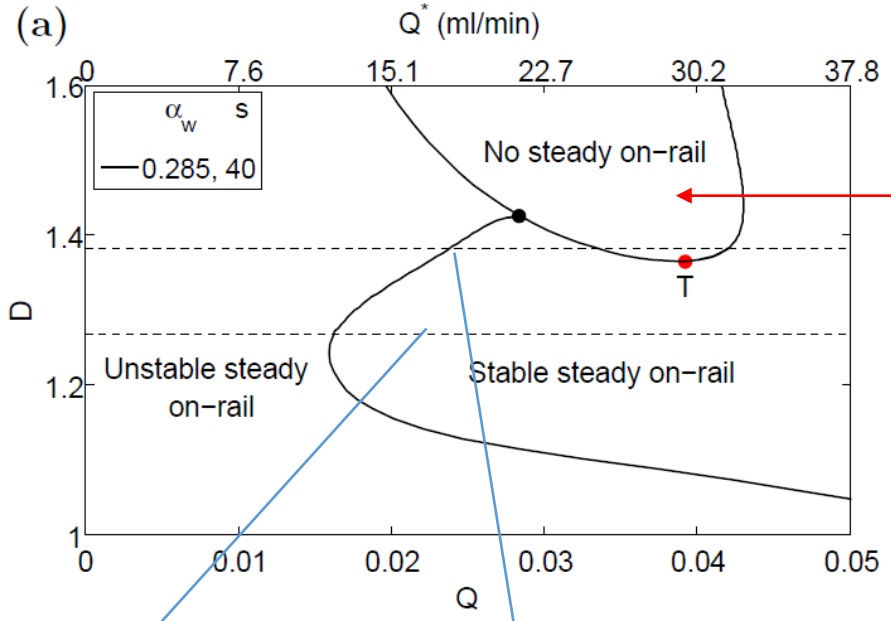


Franco-Gómez et al.
Soft Matter 2017

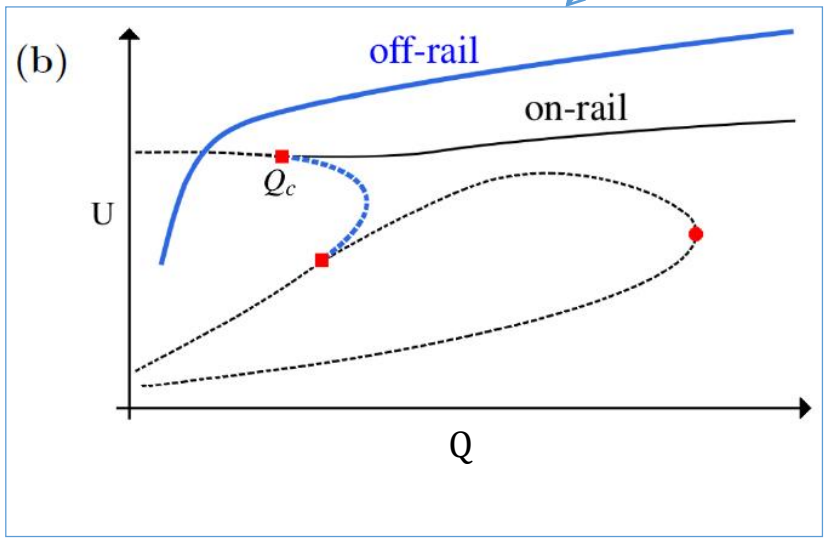


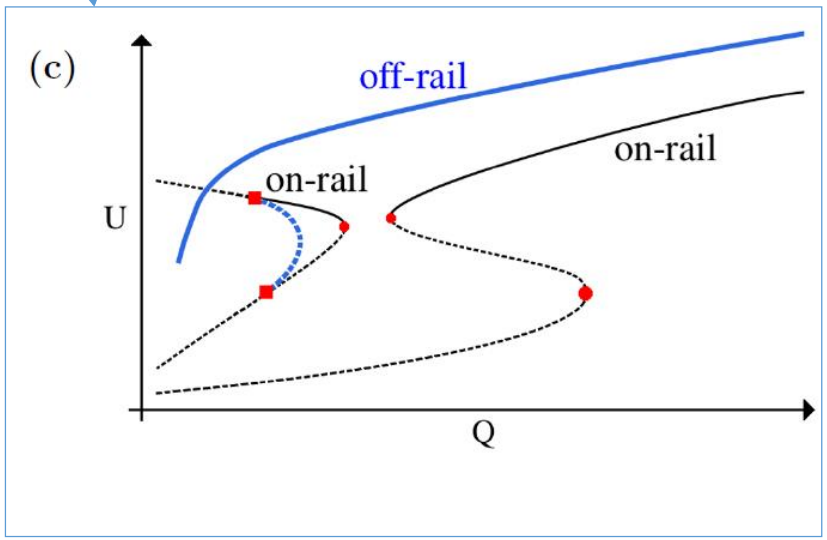


Nonlinear dynamics: numerical bifurcation diagram

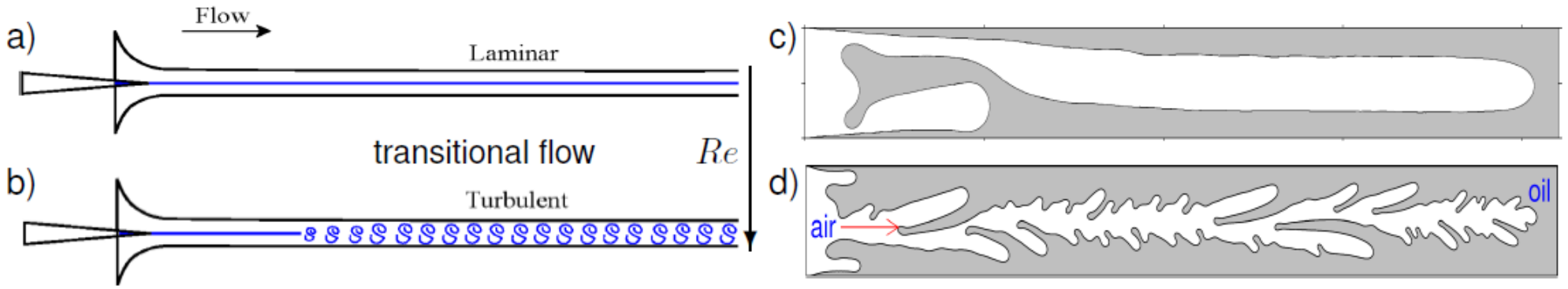


Region without a steady symmetric bubble.





Transition to disordered front propagation



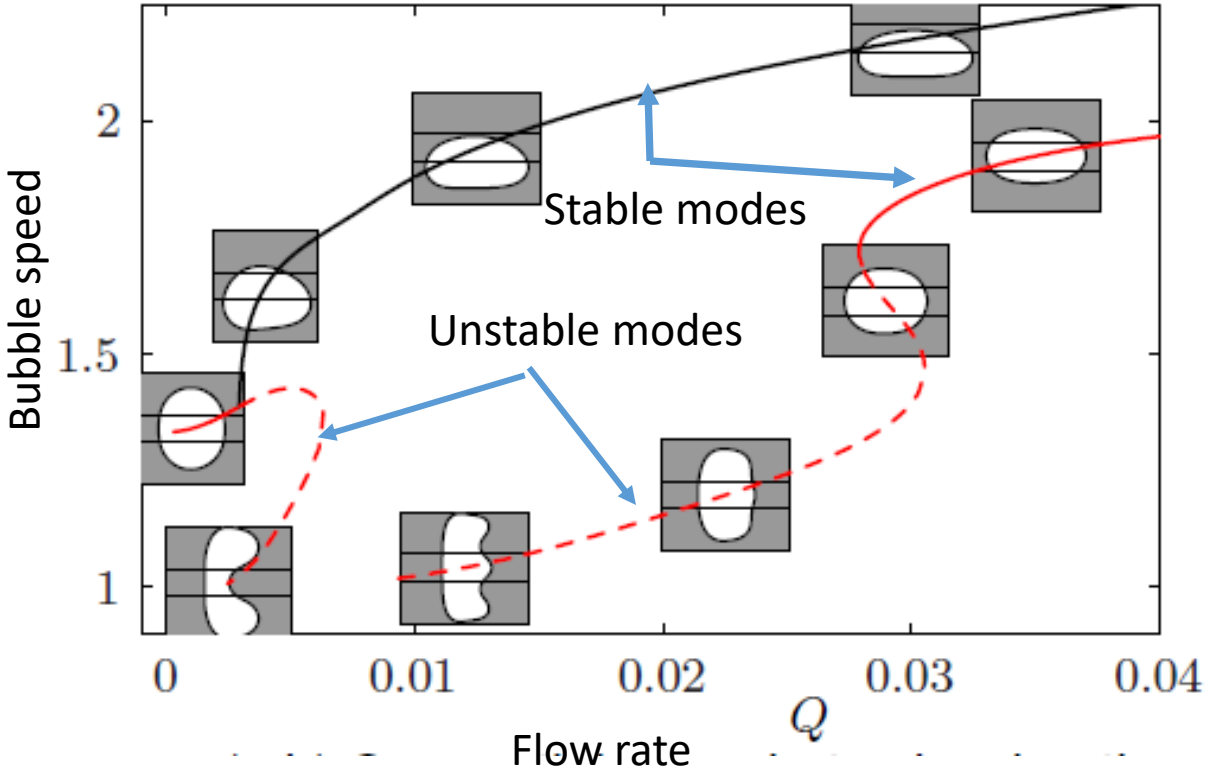
Analogy?

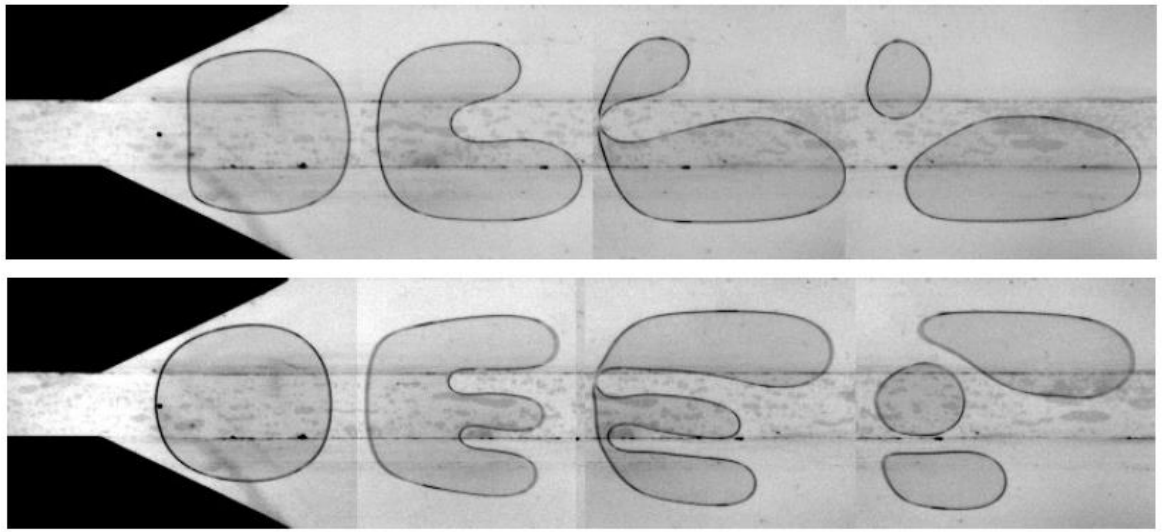
- Single finger linearly stable for all values of parameters.
- Subcritical transition at finite parameter values: finite amplitude perturbations required.

Are the weakly unstable modes of propagation key to the transition to disordered front propagation?

But much simpler than shear flow transition to turbulence!

- Steady / unsteady modes of propagation can be “easily” calculated.
- Unsteady states can be tracked experimentally: edge states?
- Work in progress…





Bubbles explore weakly unstable multiple tip solutions before breaking up.

Summary

- Multiple modes of bubble propagation in Hele-Shaw channels.
- Topography can be used to stabilise modes that are otherwise unstable in classical system.
- Applications of these nonlinear dynamics: - passive bubble actuation
  - transition to disordered front propagation.

- Can compliance be used as a means to control instabilities? Juel et al. Annu. Rev. Fluid Mech. 50 (2018)

Rigid cell

$Q^* = 145$ mL·min^{-1}

Dendritic fingers

Elastic cell: latex

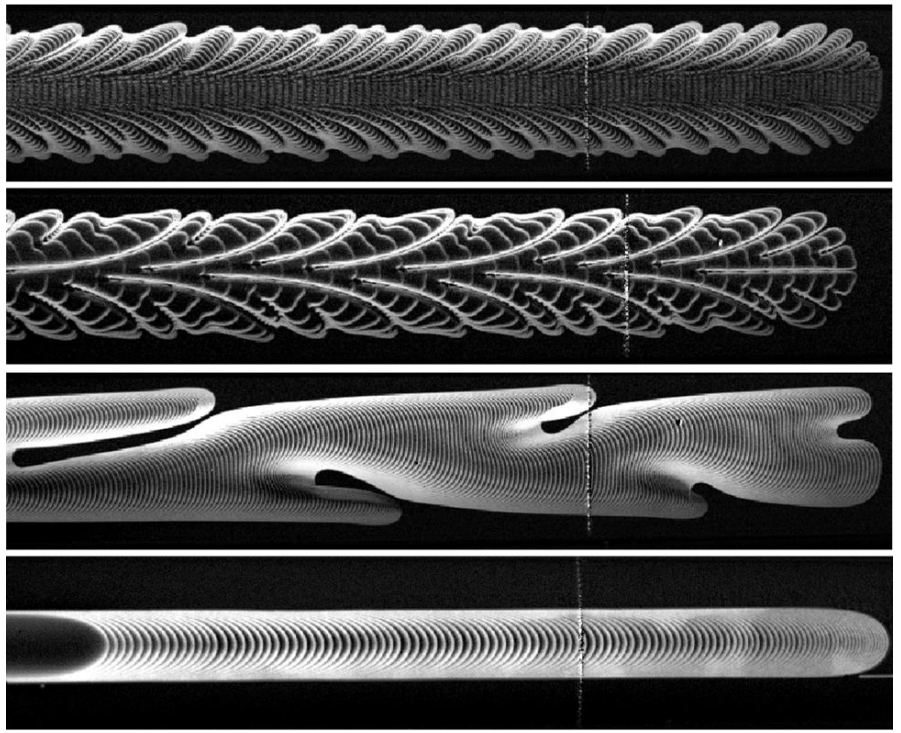
$Q^* = 145$ mL·min^{-1}

Suppressing fingers

Elastic cell: latex

$Q^* = 1250$ mL·min^{-1}

Stubby fingers



Fluids and soft materials

Anne Juel (anne.juel@manchester.ac.uk)

- *Physics of Fluids and Soft Matter group* under the umbrella of the *Manchester Centre for Nonlinear Dynamics*.
- **Powerful combination of theory and experiment.**
- From **curiosity-driven** research to the study of **practically-relevant** phenomena.
- Strong **cross-disciplinary** engagement.

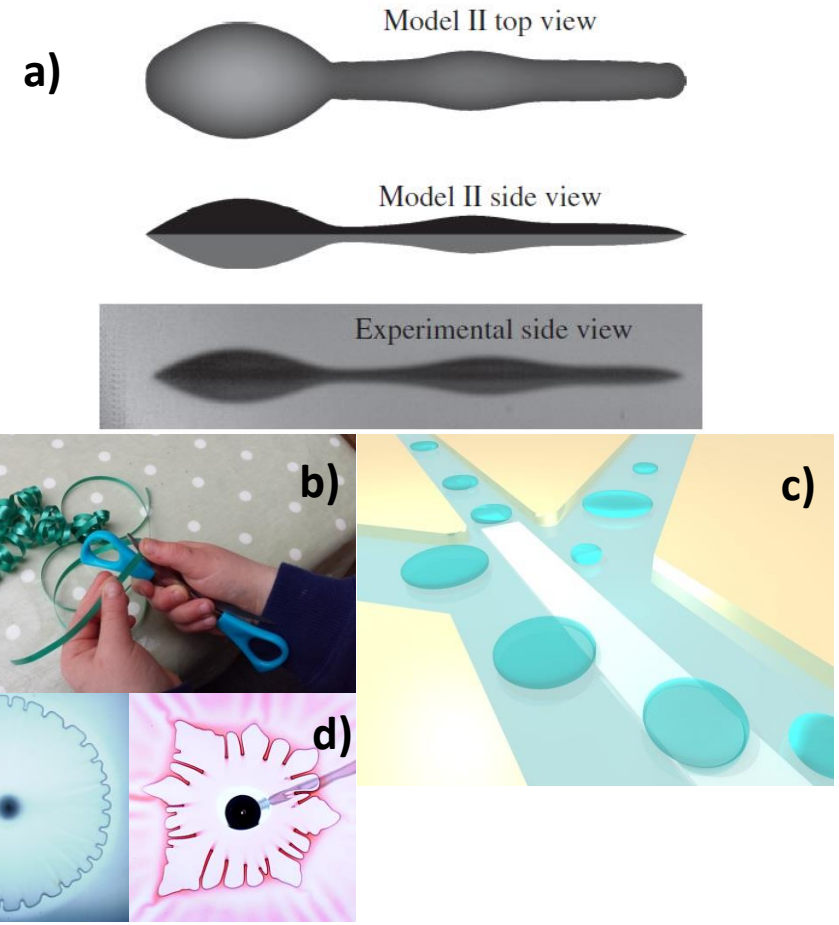
***Research interests*:**

a) **Wetting**: External control of wetting properties via electrowetting on 2D materials, additive manufacturing via inkjet printing.

b) **Complex fluids and soft solids:** Curvature generation in polymer films, fluidisation and yield phenomena in gels and dense suspensions.

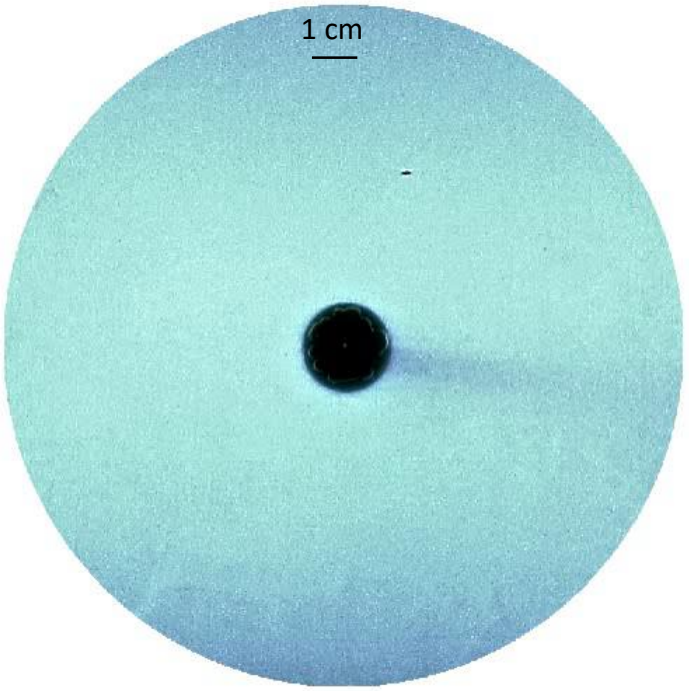
c) **Microfluidics**: Passive actuation of droplets and particles.

d) **Interaction between fluids and elastic materials**: Blistering, deformation of graphene flakes in suspension.



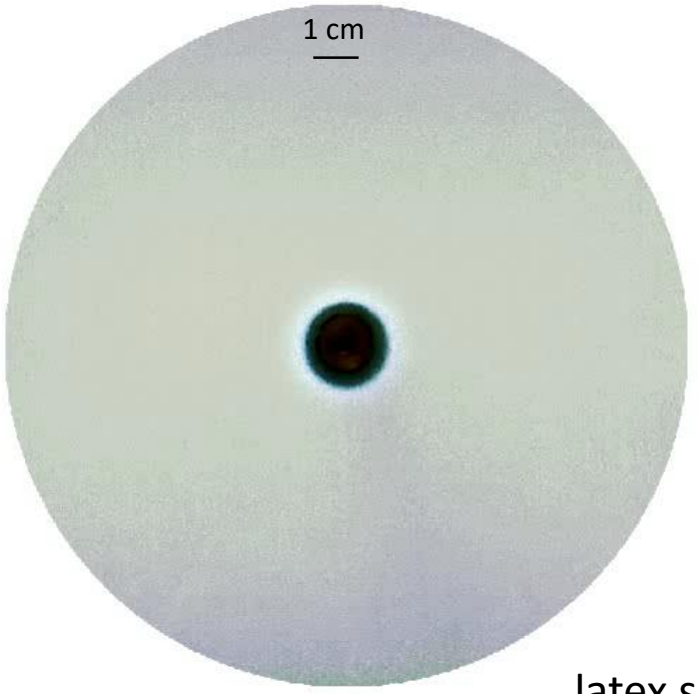
Viscous fingering

Rigid cell



top view

Elastic cell



speed x 2

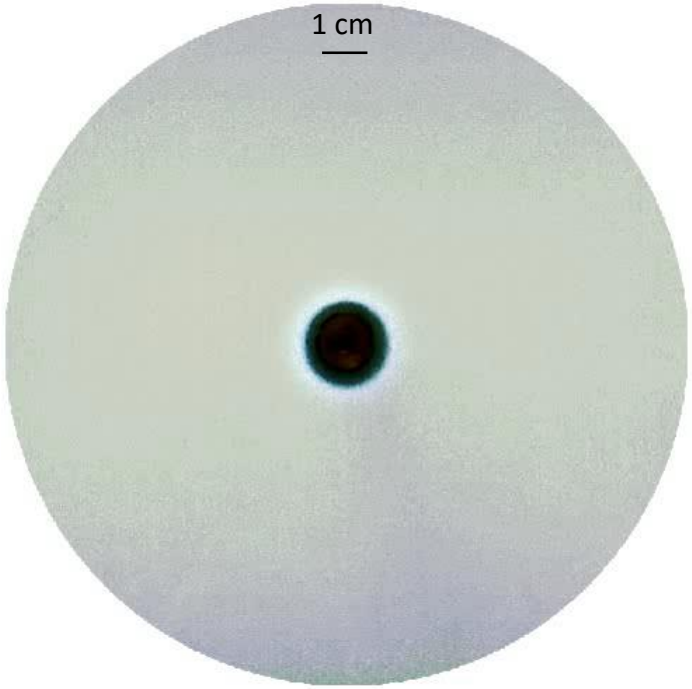
latex sheet

Flow rate : Q= 145 mL·min^{-1}

Oil viscosity : μ = 1040 kg m^{-1} s^{-1} (T=20.3°C)

Viscous fingering under elastic membranes

Q = 145 mL·min^{-1}



speed x 2

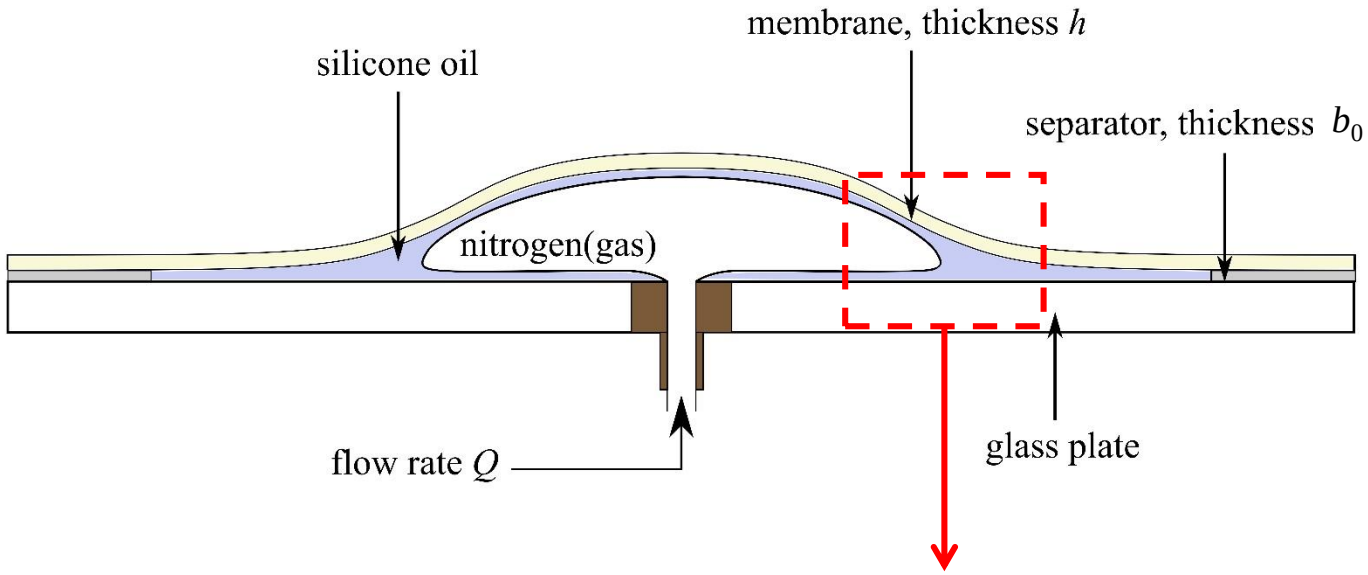
Q = 1250 mL·min^{-1}

top view

Oil viscosity : μ = 1040 kg m^{-1} s^{-1} (T=20.3°C)

Suppression mechanism

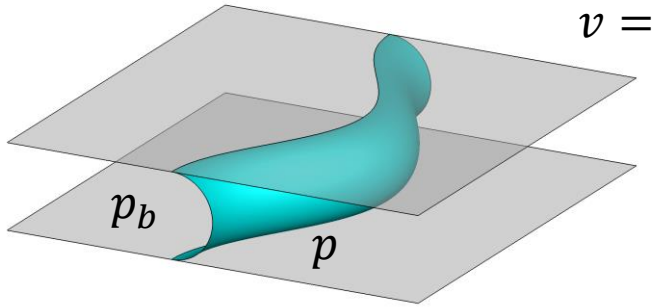
Fluid-structure interaction: Injected air works against viscous, capillary and elastic forces.

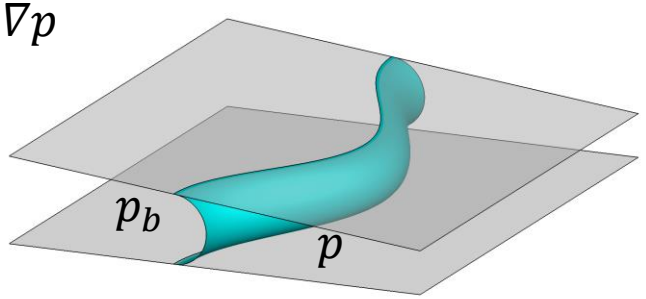


Constant depth cell

Tapered cell

$$v = -\frac{b^2}{12\,\mu}\nabla p$$





$b = b_0$ (constant)

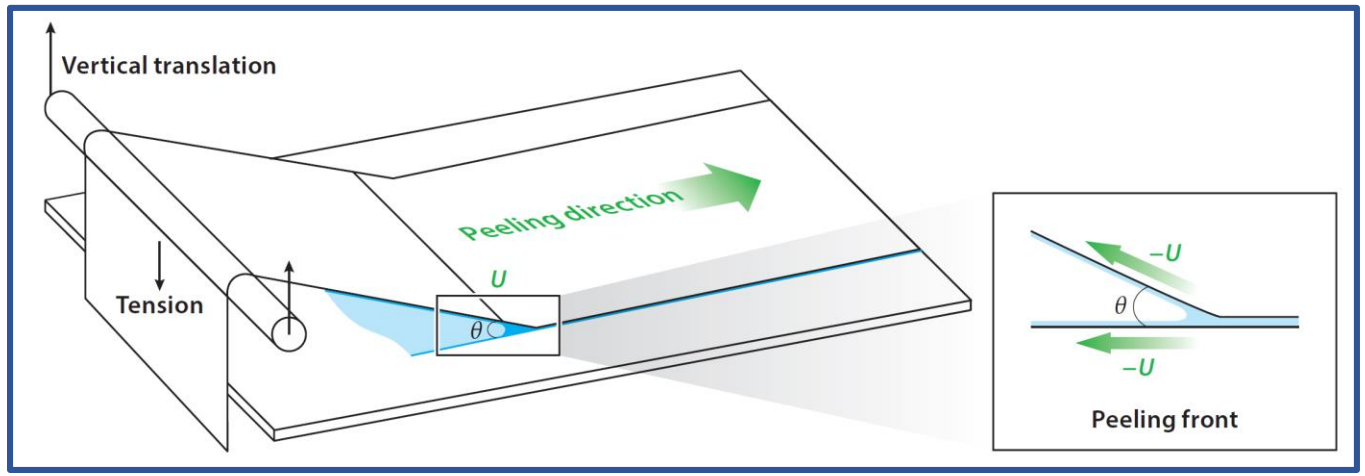
$p = p_B - Ca^{-1}(\ \kappa_{||}\ )$

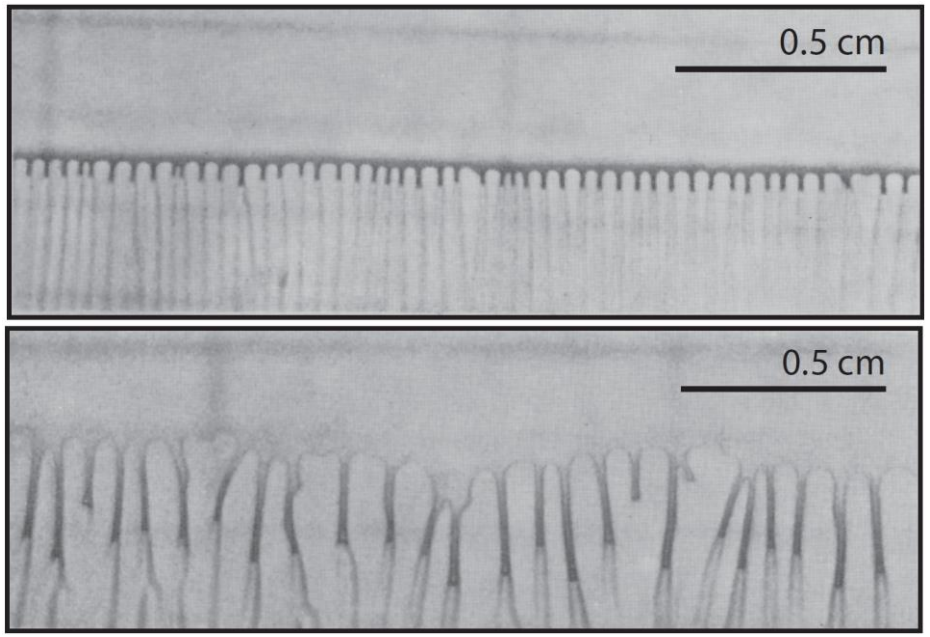
b decreases $p = p_B - Ca^{-1}(\ \kappa_{\perp} + \kappa_{||}\ )$

Pihler-Puzović et al. JFM 2013, Al-Housseiny et al Nat. Phys. (2012)

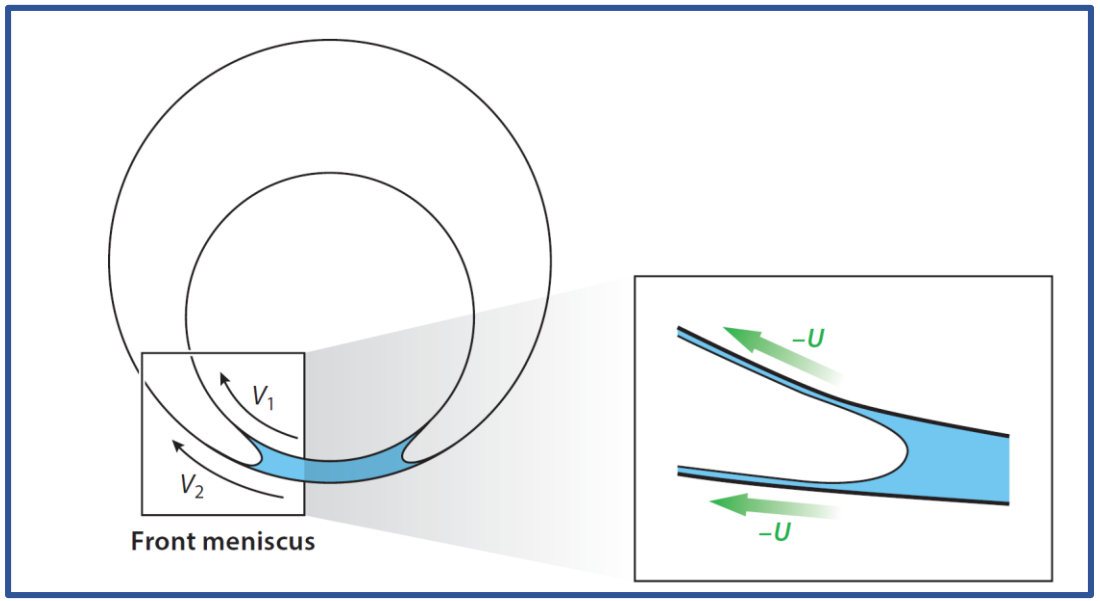
Mechanism: Printer's instability

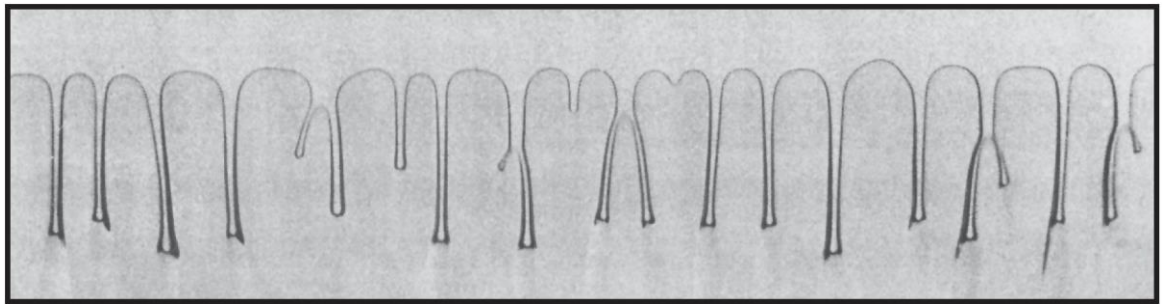
McEwan & Taylor (1966)





Couder 2000



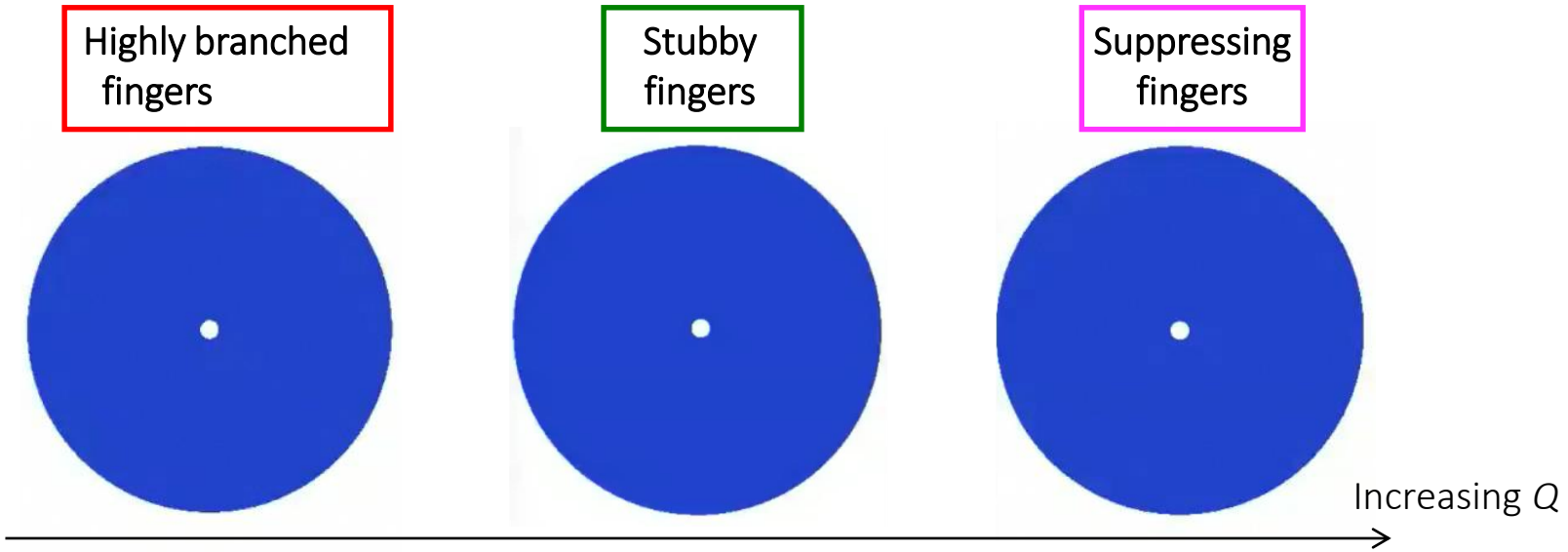


Numerical model

$$Q = \frac{12\,\mu Q^*}{2\pi A^3 K} = \frac{viscous\ forces}{plate\ bending\ stresses}$$

oomph-lib Heil & Hazel 2006

the object-oriented multi-physics finite-element library



Pihler-Puzović et al. 2013, 2018

Quantitative agreement with experiment:

- Mechanisms
- Predictive power